Mathematical Morphology And Applications
EndTerm Exam

B.Math(Hons.) III Year

November 12, 2015

Instructions: Answer as much as you can. Maximum time allotted is 3 hours. The maximum you can score is 50 marks. Marks corresponding to each question is indicated in bold.

(1) Let $\delta^\bullet : \mathcal{G}^\times \to \mathcal{G}^\bullet$ be an operator such that $\left(\delta^\bullet(X^\times), X^\times\right) = \sqcap \mathcal{G}_{X^\times}$. And, $\delta^\times : \mathcal{G}^\bullet \to \mathcal{G}^\times$ be an operator such that $\left(\overline{X^\bullet}, \overline{\delta^\bullet(X^\bullet)}\right) = \sqcup \mathcal{G}_{\overline{X^\bullet}}$. Also let $\delta = \delta^\times \circ \delta^\bullet$. Show that

(a) $\delta^\bullet(X^\times) = \left\{x \in \mathbb{G}^\bullet \mid \exists e_{x,y} \in X^\times\right\}$ for $X^\times \subseteq \mathbb{G}^\times$

(b) $\delta^\times(X^\bullet) = \left\{e_{x,y} \in \mathbb{G}^\times \mid x \in X^\bullet \ or \ y \in X^\bullet\right\}$ for $X^\bullet \subseteq \mathbb{G}^\bullet$

(c) $\delta(X^\bullet) = \left\{x \in \mathbb{G}^\bullet \mid \exists e_{x,y} \in \mathbb{G}^\times, e_{x,y} \cap X^\bullet \neq \emptyset\right\}$.

(5 + 5 + 2)

(2) Let $\{ASF_i\}_{i\geq 1}$ denote the sequence of closing alternating sequential filters with primitive structuring element being a ball of radius 1 in the continuous domain. Prove or disprove that given a binary image X, $ASF_i(X) \supseteq ASF_j(X)$ where $i \geq j$.

(5)

(3) Consider the following image under 4 connectivity in $\mathbb{Z}^2$.

2	3	6	2
3	6	3	6
255	7	6	4
2	255	7	6
1	2	255	5

(a) Define the term ‘pass value’ in the context of watersheds.

(b) Illustrate by implementing Vincent-Soille watershed algorithm that the pass values are not preserved.

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(c) Obtain the topological watershed by implementing the algorithm of your choice and verify that the pass values are preserved.

(2 + 5 + 5)

(4) Let X, Y be two sets in $\mathbb{R}^2$. Let $M^s(X,Y) = \bigcup M^s_n(X,Y)$ where

$$M^s_n(X,Y) = ((X \cap Y) \oplus nB) \cap ((X \cup Y) \ominus nB) \tag{1}$$

and $M^i(X,Y) = \bigcap M^i_n(X,Y)$ where

$$M^i_n(X,Y) = ((X \cap Y) \oplus nB) \cup ((X \cup Y) \ominus nB) \tag{2}$$

Show that $M^s(X,Y) \subseteq M^i(X,Y)$.

(8)

(5) Given a binary image on a four-connected grid. Give an algorithm to find out the number of connected components using

(a) Hit-or-miss transform (Assume that there are no holes in the connected components.)

(b) Geodesic reconstruction.

Also Justify the correctness of your algorithms.

(7 + 7)

(6) Define white top-hat transformation and black top-hat transformation. Give an application of such transformations.

(1 + 1 + 2)

(7) Assume that the domain of the image is $\mathbb{R}^2$. Take the seeds to be at $(0,0)$, $(0,1)$, and $(1,0)$ with the weights as 1, 2 and 3 respectively. Determine the weighted zone of influence for each of these points.

(7)